AP Calculus AB

Appox. and Definite Integrals Quiz Review 3

Name Key

n-Calculator:

Evaluate the following definite integrals:

1. $\int_1^4 (6x^2 - 8x + 3)\, dx$

$2x^3 - 4x^2 + 3x \Big|_1^4$

$[2(4)^3 - 4(4)^2 + 3(4)] - [2(1)^3 - 4(1)^2 + 3(1)]$

$[128 - 64 + 12] - [2 - 4 + 3]$

$76 - 1$

75

2. $\int_1^5 \frac{3x}{\sqrt{2x^2+5}}\, dx$

$u = 2x^2 + 5$

$\frac{du}{dx} = 4x$

$dx = \frac{du}{4x}$

$\int_7^{55} \frac{3x}{u^{1/2}} \cdot \frac{du}{4x}$

$\frac{3}{4} \int_7^{55} u^{-1/2}\, du$

$\frac{3}{4} \cdot 2u^{1/2} \Big|_7^{55}$

$\frac{3}{2}\left[\sqrt{55} - \sqrt{7}\right]$

3. $\int_0^{\frac{\pi}{3}} \left(\csc(2x) - \csc(2x)\cos^2(2x)\right) dx$

$\int_0^{\frac{\pi}{3}} \csc(2x)(1 - \cos^2(2x))\, dx$

$\int_0^{\frac{\pi}{3}} \csc(2x) \sin^2(2x)\, dx$

$\int_0^{\frac{\pi}{3}} \sin(2x)\, dx$

$u = 2x$

$dx = \frac{du}{2}$

$\frac{1}{2}\int_0^{\frac{2\pi}{3}} \sin u\, du$

$-\frac{1}{2}\cos u \Big|_0^{\frac{2\pi}{3}}$

$-\frac{1}{2}\left(\cos\frac{2\pi}{3} - \cos 0\right) = -\frac{1}{2}\left(-\frac{1}{2} - 1\right) = \frac{3}{4}$

4. $\int_2^5 \frac{3x}{x^2 - 3}\, dx$

$u = x^2 - 3$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int_1^{22} \frac{3x}{u} \cdot \frac{du}{2x}$

$\frac{3}{2}\int_1^{22} \frac{1}{u}\, du$

$\frac{3}{2} \ln|u| \Big|_1^{22}$

$\frac{3}{2}[\ln 22 - \ln 1]$

$\frac{3}{2}\ln 22$

5. Given $\frac{dy}{dx} = \frac{2x + 4x^3}{y}$ and $f(2) = -6$, find $f(x)$.

$\int y\, dy = \int 2x + 4x^3\, dx$

$\frac{1}{2}y^2 = x^2 + x^4 + C$

$\frac{1}{2}(-6)^2 = (2)^2 + (2)^4 + C$

$18 = 4 + 16 + C$

$-2 = C$

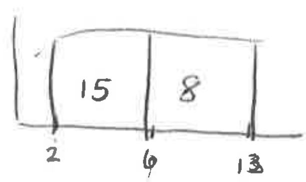
$2\left(\frac{1}{2}y^2 = x^4 + x^2 - 2\right)$

$y^2 = 2x^4 + 2x^2 - 4$

$y = \pm\sqrt{2x^4 + 2x^2 - 4}$

$y = -\sqrt{2x^4 + 2x^2 - 4}$ b/c $f(2) = -6$

6. Given: $\int_2^6 g(x)\, dx = 15$ and $\int_6^{13} g(x)\, dx = 8$ and that the function is continuous, differentiable and greater than zero on $(-\infty, \infty)$, find the following values:



a) $\int_{13}^{6} g(x)\, dx = -8$

b) $\int_2^{13} g(x)\, dx = 23$

c) $3\int_2^6 g(x)\, dx = 45$

d) $2 - \int_{13}^{2} g(x)\, dx = 2 - {-23} = 25$

7. A particle is moving along a horizontal path such that its velocity is given by the function $v(t) = t^2 - 10t + 16$. Set up an integral expression and use a graphing calculator to evaluate the integral that will give you the following:

a) The total distance traveled in the first 12 seconds.

$$\int_0^{12} |v(t)|\, dt = 120 \text{ ft.}$$

b) The displacement in the first 12 seconds.

$$\int_0^{12} v(t)\, dt = 48 \text{ ft.}$$

8. Given $f(x) = 4 + \sqrt{x-2}$, approximate the area bound by $f(x)$, the lines $x = 2$, $x = 12$ and the x-axis if $n = 5$

$$\Delta x = \frac{12-2}{5} = 2$$

a) Right endpoints b) Left endpoints c) Midpoints d) Trapezoids

$x_0 = 2$ $x_1 = 4$ $x_2 = 6$ $x_3 = 8$ $x_4 = 10$ $x_5 = 12$

a) $R_5 = (2)[f(4) + f(6) + \cdots + f(12)] = 63.709$

b) $L_5 = (2)[f(2) + f(4) + \cdots + f(10)] = 57.384$

c) $M_5 = (2)[f(3) + f(5) + \cdots + f(11)] = 61.228$

d) $T_5 = \frac{(2)}{(2)}[f(2) + 2f(4) + 2f(6) + 2f(8) + 2f(10) + f(12)] = 60.547$

9. The rate at which factory B produces jellybeans is modeled by the function $r(t) = 80 + 5\sqrt{x+1}$ where *t* is time in hours since the factory opens and *r(t)* is measured in pounds of jellybeans. The factory has to fill 31 pound bags for shipping to regional distributors. On Tuesday, April 5th, the factory closes at 5:00 pm and has 27 pounds of jellybeans left unpacked in a shipping bag. The next day, they open at 9:00 am and close at 5:00 pm. How many total bags (31 pounds each) were they able to ship out on Wednesday, April 6th?

$$27 + \int_0^8 r(t)\, dt = 753.667 \text{ pounds}$$

$$\frac{753.667 \text{ pds.}}{31} = 24.3118 \text{ bags}$$